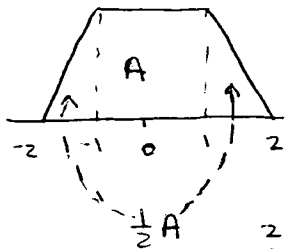Uitwerking 14-10-'96

① ⓐ normeer $\Rightarrow$ $\int_a^b f(x)\,dx = 1$

$A\int_{-2}^{-1}(x+2)\,dx + A\int_{-1}^{1} 1\,dx + A\int_{1}^{2}(-x+2) = 1 \quad \Rightarrow \quad A = \frac{1}{3}$

handig : oppervlakte grafiek moet 1 zijn :

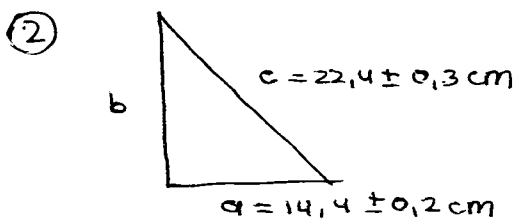$opp = 3A = 1 \Rightarrow A = \frac{1}{3}$

ⓑ $\bar{x} = A\int_{-2}^{2} x\,f(x)\,dx = 0$ (symmetrie !)

ⓒ $\overline{x^2} = A\int_{-2}^{-1} x^2(x+2)\,dx + A\int_{-1}^{1} x^2\,dx + A\int_{1}^{2}x^2(-x+2)\,dx = 2\frac{1}{2}A = \frac{5}{6}$

$\overline{x}^2 = (\bar{x})^2 = 0$ (zie ⓑ)

$\sigma^2 = \overline{x^2} - \bar{x}^2 \quad \Rightarrow \quad \sigma = \sqrt{\frac{5}{6} - 0} = \sqrt{\frac{5}{6}}$

ⓓ oppervlakte $(1,2) = \frac{1}{2}A = \frac{1}{6}$ = kans op waarde in interval $(1,2)$
anders : $P\{1 \le x \le 2\} = \int_1^2 f(x)\,dx = A\int_1^2 (-x+2)\,dx$

②

$b = \sqrt{c^2 - a^2}$

Omtrek $= a + b + c = a + c + \sqrt{c^2 - a^2}$

$S_{omtrek}^2 = \left(\frac{\partial Omtrek}{\partial c}\right)^2 S_c^2 + \left(\frac{\partial Omtrek}{\partial a}\right)^2 S_a^2$

$= \left(1 + \frac{c}{\sqrt{c^2 - a^2}}\right)^2 S_c^2 + \left(1 + \frac{-a}{\sqrt{c^2 - a^2}}\right)^2 S_a^2$

$b = 17,2 \pm 0,7$ cm

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(3) (a) $M = \sum w_i D_i^2 = \text{Minimaal} = \sum w_i (y_i - ax_i - b)^2$

$\Rightarrow$ differentiëren naar a : $a \sum w_i x_i^2 + b \sum w_i x_i - \sum w_i x_i y_i = 0$

"   " b : $a \sum w_i x_i + b \sum w_i - \sum w_i y_i = 0$

$\Rightarrow \quad a = \dfrac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - (\bar{x})^2} \quad \text{met} \quad \bar{x} = \dfrac{\sum w_i x_i}{\sum w_i} \quad \text{en} \quad w_i = \dfrac{1}{s_i^2}$

$\qquad b = \bar{y} - a\bar{x}$

(b) $\bar{x} = \dfrac{1+2+3+4+5}{5} = 3$ (alle $w_i = 1$, want fout in x_i verwaarlozen

$\bar{y} = \dfrac{1 \cdot 1 + 2 \cdot 1 + 4 \cdot 4 + 5 \cdot 4 + 6 \cdot 4}{1 + 1 + 4 + 4 + 4} = 4\frac{1}{2}$

$\overline{xy} = \dfrac{\sum w_i x_i y_i}{\sum w_i} = \dfrac{1 \cdot 1 \cdot 1 + 2 \cdot 2 \cdot 1 + 3 \cdot 4 \cdot 4 + 4 \cdot 5 \cdot 4 + 5 \cdot 6 \cdot 4}{14} = \dfrac{253}{14}$

$\overline{x^2} = \dfrac{1+4+9+16+25}{5} = 11$

$a = \dfrac{\frac{253}{14} - 3 \cdot 4\frac{1}{2}}{11 - 3^2} = \dfrac{4\frac{4}{7}}{2} \cong 2{,}3$

$b \cong 4\frac{1}{2} - 2{,}3 \cdot 3 \cong -2{,}4$